

නව නිර්දේශය/புதிய பாடத்திட்டம்/New Syllabus

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
 திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம்
 Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka
 இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம்
 Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
 General Certificate of Education (Adv. Level) Examination, 2020

සංයුක්ත ගණිතය I
 இணைந்த கணிதம் I
 Combined Mathematics I

10 E I

පැය තුනයි
 மூன்று மணித்தியாலம்
 Three hours

අමතර කියවීමේ කාලය - මිනිත්තු 10 යි
 மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
 Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number

Instructions:

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Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17).
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- * **Part B:**
 Answer **five** questions only. Write your answers on the sheets provided.
- * At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- * You are permitted to remove **only Part B** of the question paper from the Examination Hall.

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
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B	11	
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	13	
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	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner	
Checked by:	1
	2
Supervised by:	

7. Show that the equation of the normal line to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ at the point $P \equiv (5 \cos \theta, 3 \sin \theta)$ on it, is $5 \sin \theta x - 3 \cos \theta y = 16 \sin \theta \cos \theta$.

Find the y -intercept of the normal line drawn to the above ellipse at the point $\left(\frac{5}{2}, \frac{3\sqrt{3}}{2}\right)$ on it.

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8. Let $m \in \mathbb{R}$ and l be the straight line passing through the point $A \equiv (1, 2)$ with gradient m . Write down the equation of l in terms of m . It is given that the perpendicular distance from the point $B \equiv (2, 3)$ to the line l is $\frac{1}{\sqrt{5}}$ units. Find the values of m .

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13.(a) Let $\mathbf{A} = \begin{pmatrix} a+1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ a & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} a & 1 \\ a & 2 \end{pmatrix}$, where $a \in \mathbb{R}$.

Show that $\mathbf{A}^T \mathbf{B} - \mathbf{I} = \mathbf{C}$; where \mathbf{I} is the identity matrix of order 2.

Show also that \mathbf{C}^{-1} exists **if and only if** $a \neq 0$.

Now, let $a = 1$. Write down \mathbf{C}^{-1} .

Find the matrix \mathbf{P} such that $\mathbf{CPC} = 2\mathbf{I} + \mathbf{C}$.

(b) Let $z, w \in \mathbb{C}$. Show that $|z|^2 = z\bar{z}$ and applying it to $z - w$,

$$\text{show that } |z - w|^2 = |z|^2 - 2\operatorname{Re} z\bar{w} + |w|^2.$$

Write a similar expression for $|1 - z\bar{w}|^2$ and show that $|z - w|^2 - |1 - z\bar{w}|^2 = -(1 - |z|^2)(1 - |w|^2)$.

Deduce that if $|w| = 1$ and $z \neq w$, then $\left| \frac{z - w}{1 - z\bar{w}} \right| = 1$.

(c) Express $1 + \sqrt{3}i$ in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $0 < \theta < \frac{\pi}{2}$.

It is given that $(1 + \sqrt{3}i)^m (1 - \sqrt{3}i)^n = 2^8$, where m and n are positive integers.

Applying De Moivre's theorem, obtain equations sufficient to determine the values of m and n .

14.(a) Let $f(x) = \frac{x(2x-3)}{(x-3)^2}$ for $x \neq 3$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \frac{9(1-x)}{(x-3)^3}$ for $x \neq 3$.

Hence, find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

Also, find the coordinates of the turning point of $f(x)$.

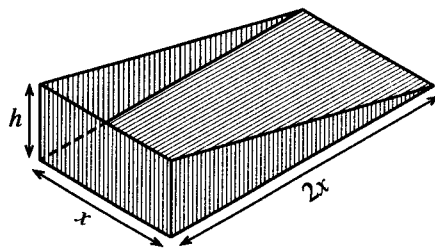
It is **given that** $f''(x) = \frac{18x}{(x-3)^4}$ for $x \neq 3$.

Find the coordinates of the point of inflection of the graph of $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection.

(b) The adjoining figure shows the portion of a dust pan without its handle. Its dimensions in centimetres, are shown in the figure. It is given that its volume $x^2 h \text{ cm}^3$ is 4500 cm^3 .

Its surface area $S \text{ cm}^2$ is given by $S = 2x^2 + 3xh$. Show that S is minimum when $x = 15$.



15.(a) It is given that there exist constants A and B such that

$$x^3 + 13x - 16 = A(x^2 + 9)(x + 1) + B(x^2 + 9) + 2(x + 1)^2 \text{ for all } x \in \mathbb{R}.$$

Find the values of A and B .

Hence, write down $\frac{x^3 + 13x - 16}{(x + 1)^2 (x^2 + 9)}$ in partial fractions and

$$\text{find } \int \frac{x^3 + 13x - 16}{(x + 1)^2 (x^2 + 9)} dx .$$

(b) Using integration by parts, evaluate $\int_0^1 e^x \sin^2 \pi x dx$.

(c) Using the formula $\int_0^a f(x) dx = \int_0^a f(a - x) dx$, where a is a constant,

$$\text{show that } \int_0^\pi x \cos^6 x \sin^3 x dx = \frac{\pi}{2} \int_0^\pi \cos^6 x \sin^3 x dx.$$

$$\text{Hence, show that } \int_0^\pi x \cos^6 x \sin^3 x dx = \frac{2\pi}{63}.$$

16. Let $A \equiv (1, 2)$ and $B \equiv (3, 3)$.

Find the equation of the straight line l passing through the points A and B .

Find the equations of the straight lines l_1 and l_2 passing through A , each making an acute angle $\frac{\pi}{4}$ with l .

Show that the coordinates of any point on l can be written in the form $(1 + 2t, 2 + t)$, where $t \in \mathbb{R}$.

Show also that the equation of the circle C_1 lying entirely in the first quadrant with radius $\frac{\sqrt{10}}{2}$, touching both l_1 and l_2 , and its centre on l is $x^2 + y^2 - 6x - 6y + \frac{31}{2} = 0$.

Write down the equation of the circle C_2 whose ends of a diameter are A and B .

Determine whether the circles C_1 and C_2 intersect orthogonally.

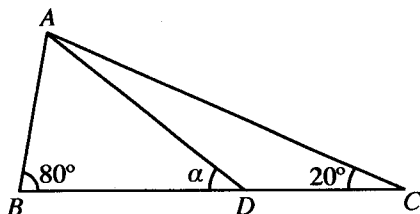
17.(a) Write down $\sin(A-B)$ in terms of $\sin A$, $\cos A$, $\sin B$ and $\cos B$.

Deduce that

(i) $\sin(90^\circ - \theta) = \cos \theta$, and

(ii) $2 \sin 10^\circ = \cos 20^\circ - \sqrt{3} \sin 20^\circ$.

(b) In the usual notation, state the **Sine Rule** for a triangle ABC .



In the triangle ABC shown in the figure, $\hat{A}BC = 80^\circ$ and $\hat{A}CB = 20^\circ$. The point D lies on BC such that $AB = DC$. Let $\hat{A}DB = \alpha$.

Using the **Sine Rule** for suitable triangles, show that $\sin 80^\circ \sin(\alpha - 20^\circ) = \sin 20^\circ \sin \alpha$.

Explain why $\sin 80^\circ = \cos 10^\circ$ and **hence**, show that $\tan \alpha = \frac{\sin 20^\circ}{\cos 20^\circ - 2 \sin 10^\circ}$.

Using the result in (a)(ii) above, **deduce** that $\alpha = 30^\circ$.

(c) Solve the equation $\tan^{-1}(\cos^2 x) + \tan^{-1}(\sin x) = \frac{\pi}{4}$.

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ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
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 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
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සංයුක්ත ගණිතය II
 இணைந்த கணிதம் II
 Combined Mathematics II

10 E II

පැය තුනයි
 மூன்று மணித்தியாலம்
 Three hours

අමතර කියවීමේ කාලය - මිනිත්තු 10 යි
 மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
 Additional Reading Time - 10 minutes

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- * You are permitted to remove **only Part B** of the question paper from the Examination Hall.
- * In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	Question No.	Marks
A	1	
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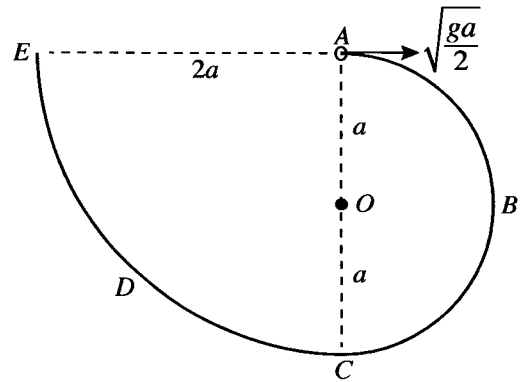
Total

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Checked by:	1
	2
Supervised by:	

(b) A smooth thin wire $ABCDE$ is fixed in a vertical plane, as shown in the figure. The portion ABC is a semicircle with centre O and radius a , and the portion CDE is a quarter of a circle with centre A and radius $2a$. The points A and C lie on the vertical line through O and the line AE is horizontal. A small smooth bead P of mass m is placed at A and is given a velocity $\sqrt{\frac{ga}{2}}$ horizontally, and begins to move along the wire.

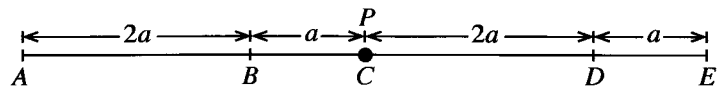


Show that the speed v of the bead P when \overrightarrow{OP} makes an angle θ ($0 \leq \theta \leq \pi$) with \overrightarrow{OA} is given by $v^2 = \frac{ga}{2}(5 - 4\cos\theta)$.

Find the reaction on the bead P from the wire at the above position and show that it changes its direction when the bead P passes the point $\theta = \cos^{-1}\left(\frac{5}{6}\right)$.

Write down the velocity of the bead P just before it leaves the wire at E and find the reaction on the bead P from the wire at that instant.

13. The points A, B, C, D and E lie on a straight line in that order, on a smooth horizontal table such that $AB = 2a$, $BC = a$, $CD = 2a$ and $DE = a$, as shown in the figure. One end of a light elastic string of natural length $2a$ and modulus of elasticity kmg is attached to the point A and the other end to a particle P of mass m . One end of another light elastic string of natural length a and modulus of elasticity mg is attached to the point E and the other end to the particle P . When the particle P is held at C and released, it stays in equilibrium. Find the value of k .



Now, the string AP is pulled until the particle P reaches the point D and released from rest. Show that the equation of motion of P from D to B is given by $\ddot{x} + \frac{3g}{a}x = 0$, where $CP = x$.

Using the formula $\dot{x}^2 = \frac{3g}{a}(c^2 - x^2)$, where c is the amplitude, show that the velocity of particle P when it reaches B is $3\sqrt{ga}$.

An impulse is given to the particle P when it reaches B so that the velocity of P just after the impulse is \sqrt{ag} in the direction of \overrightarrow{BA} .

Show that the equation of motion of P after passing B until it comes to instantaneous rest is given by $\ddot{y} + \frac{g}{a}y = 0$, where $DP = y$.

Show that the total time taken by the particle P , started at D , to reach B for the second time is

$$2\sqrt{\frac{a}{g}} \left(\frac{\pi}{3\sqrt{3}} + \cos^{-1}\left(\frac{3}{\sqrt{10}}\right) \right).$$

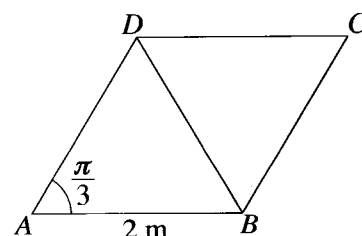
14. (a) Let \mathbf{a} and \mathbf{b} be two **unit vectors**.

The position vectors of three points A, B and C with respect to an origin O , are $12\mathbf{a}$, $18\mathbf{b}$ and $10\mathbf{a} + 3\mathbf{b}$ respectively. Express \overrightarrow{AC} and \overrightarrow{CB} in terms of \mathbf{a} and \mathbf{b} .

Deduce that A, B and C are collinear and find $AC : CB$.

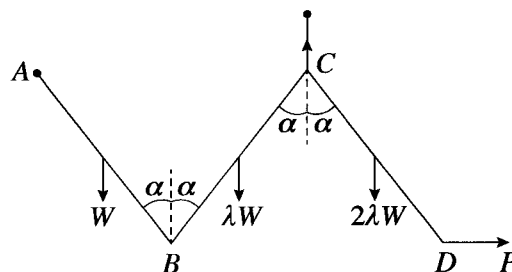
It is given that $OC = \sqrt{139}$. Show that $\widehat{AOB} = \frac{\pi}{3}$.

(b) Let $ABCD$ be a rhombus with $AB = 2$ m and $\widehat{BAD} = \frac{\pi}{3}$. Forces of magnitude 10 N, 2 N, 6 N, P N and Q N act along AD, BA, BD, DC and CB respectively, in the directions indicated by the order of the letters. It is given that the resultant force is of magnitude 10 N and its direction is in the direction parallel to BC in the sense from B to C . Find the values of P and Q . Also, find the distance from A to the point where the line of action of the resultant force meets BA produced.



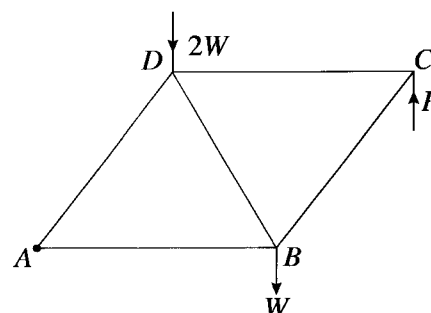
Now, a couple of moment M Nm acting in the counterclockwise sense and two forces, each of magnitude F N acting along CB and DC in the directions indicated by the order of the letters, are added to the system so that the resultant force passes through the points A and C . Find the values of F and M .

15. (a) Three uniform rods AB, BC and CD , each of length $2a$ are smoothly joined at the ends B and C . The weights of the rods AB, BC and CD are $W, \lambda W$ and $2\lambda W$, respectively. The end A is smoothly hinged to a fixed point. The rods are kept in equilibrium in a vertical plane by a light inextensible string attached to the joint C and to a fixed point vertically above C and by a horizontal force P applied to the end D such that A and C are at the same horizontal level and each of the rods making an angle α with the vertical, as shown in the figure. Show that $\lambda = \frac{1}{3}$.



Show also that the horizontal and vertical components of the force exerted on AB by CB at B are $\frac{W}{3} \tan \alpha$ and $\frac{W}{6}$, respectively.

(b) The framework shown in the adjoining figure is made from light rods AB, BC, CD, DA and BD , each of length $2a$, freely jointed at A, B, C and D . There are loads of W and $2W$ at B and D , respectively. The framework is smoothly hinged at A to a fixed point and kept in equilibrium with AB horizontal by a vertical force P applied to it at C , as shown in the figure. Find the value of P in terms of W .

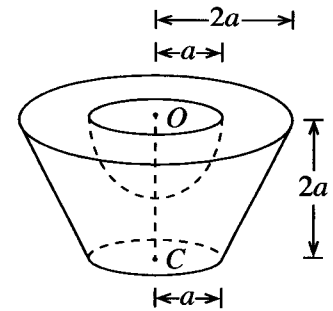


Draw a stress diagram using Bow's notation and **hence**, find the stresses in the rods stating whether they are tensions or thrusts.

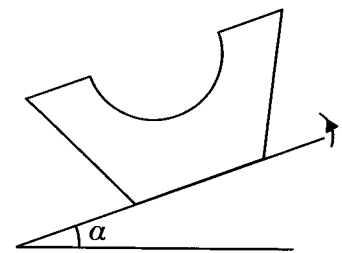
16. Show that the centre of mass of

- (i) a uniform solid right circular cone of base radius r and height h is at a distance $\frac{h}{4}$ from the centre of the base,
- (ii) a uniform solid hemisphere of radius r is at a distance $\frac{3r}{8}$ from its centre.

The adjoining figure shows a mortar S made by removing a solid hemisphere from a frustum of a solid uniform right circular cone having base radius $2a$ and height $4a$. The radius and the centre of the upper circular face of the frustum are $2a$ and O , respectively, and those for the lower circular face are a and C , respectively. The height of the frustum is $2a$. The radius and the centre of the removed solid hemisphere are a and O , respectively. Show that the centre of mass of mortar S lies at a distance $\frac{41}{48}a$ from O .



Mortar S is placed on a rough horizontal plane with its lower circular face touching the plane. Now, the plane is tilted upwards slowly. The coefficient of friction between the mortar and the plane is 0.9. Show that if $\alpha < \tan^{-1}(0.9)$, then the mortar stays in equilibrium, where α is the inclination of the plane to the horizontal.



- 17.(a) In a certain factory, machine A makes 50% of the items and the rest are made by machines B and C . It is known that 1%, 3% and 2% of the items made by A , B and C respectively are defective. The probability that a randomly selected item is defective is given to be 0.018. Find the percentages of items made by the machines B and C .
Given that a randomly selected item is defective, find the probability that it was made by the machine A .

- (b) The time taken (in minutes) to travel to work from their homes of 100 employees of a certain factory are given in the following table:

Time taken	Number of employees
0 – 20	10
20 – 40	30
40 – 60	40
60 – 80	10
80 – 100	10

Estimate the mean, standard deviation and the mode of the distribution given above.

Later, all of the employees in the class interval 80–100 moved closer to the factory. It has changed the frequency of the class interval 80–100 from 10 to 0 and the frequency of the class interval 0–20 from 10 to 20.

Estimate the mean, standard deviation and the mode of the new distribution.
